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A NEW AIRMASS INDEPENDENT FORMULATION FOR THE LINKE TURBIDITY COEFFICIENT

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Abstract—We propose a new formulation for the Linke turbidity coefficient with the objective of removing its dependence upon solar geometry. In the process, we also develop two new simple clear sky models for global and direct normal irradiance.

1. INTRODUCTION

The derivation of the ground solar radiation components on a large geographic and temporal scale requires the knowledge of the clear sky atmospheric transmittance on the same scales. This information can be obtained via utilization of well known turbidity coefficients as for example the Linke turbidity coefficient, or from radiation transfer models with the aerosols and water vapor optical thicknesses as input parameters. The former can be derived from beam radiation measurement networks and interpolated to create maps, the latter quantities are not measured in automatic networks, and hypotheses on the nature and quantity of the aerosols have to be made in order to use them as an input. In the frame of solar radiation applications, a broadband turbidity coefficient is more appropriate and easier to implement. The Linke turbidity coefficient T_L has the advantage to be widely used since 1922 (Linke, 1922) to quantify this information, but has the disadvantage to be dependent on the air mass. A number of authors have tried to circumvent this difficulty by different means. The most popular method was to normalize the measured values of T_L at air mass = 2 (Kasten, 1988; Grenier *et al*., 1994). Linke himself (1942) recognized the variation of T_L with air mass but had little success in introducing a new extinction coefficient based on an atmosphere of pure air containing 1 cm of water.

In the present paper, we develop a new formulation for the Linke turbidity factor, fully compatible with the original formulation at air mass 2, but roughly independent of the air mass. The first step in this process was to modify existing beam irradiance clear sky models in order to better take into account the radiation's dependence with the altitude of the considered station and the solar geometry. The new turbidity factor is then derived by inversion of the beam clear sky radiation model.

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2. THE LINKE TURBIDITY COEFFICIENT

2.1. Historical context

Linke (1922) proposed to express the total optical thickness of a cloudless atmosphere as the product of two terms, δ_{cda}, the optical thickness of a water- and aerosol-free atmosphere (clear and dry atmosphere), and the Linke turbidity coefficient T_L which represents the number of clean and dry atmospheres producing the observed extinction:

$$B_{nc} = I_o \cdot \exp(-\delta_{cda} \cdot T_L \cdot AM) \quad (1)$$

where B_{nc} is the normal incidence beam irradiance, I_o the normal incidence extraterrestrial irradiance and AM the altitude corrected air mass (Kasten and Young, 1989).

This definition depends on the theoretical value of δ_{cda} which is used to evaluate T_L. A careful examination of the definition of the terms δ_{cda} and T_L is helpful in getting a clear picture of Linke's formalism and the developments made since Linke first proposed it. Linke (1922) defined δ_{cda} as the integrated optical thickness of the terrestrial atmosphere free of clouds, water vapor and aerosols, which he computed from theoretical assumptions and apparently validated in a very pure, dry mountain atmosphere. He used the following formulation:

$$\delta_{\mathrm{cda}} = 0.128 - 0.054 \cdot \log(AM) \tag{2}$$

thus, T_{L} represents the number of clean dry atmospheres necessary to produce the observed attenuation, resulting from the additional and highly variable effects of water vapor and aerosols. Obviously, the smallest possible value of T_{L} at sea level should be 1. Feussner and Dubois (1930) published a series of spectral data tables enabling the calculation of δ_{cda} where both molecular scattering and absorption by the stratospheric ozone layer are taken into account. Kasten (1980) fitted the following equation to these tables:

$$\delta_{\mathrm{cda}} = (9.4 + 0.9 \cdot AM)^{-1} \tag{3}$$

which is known as Kasten's pyrheliometric formula. In this widely used relation, absorption by the permanent atmospheric gases such as CO_2, O_2, N_2O, CO, etc. are not taken into account. The effect of these gases is therefore included in the term T_{L}, incorrectly contributing to atmospheric turbidity, as noted by Katz *et al.* (1982) and confirmed by Kasten (1996). The dependence of δ_{cda} upon air mass is a consequence of the strong dependence of Rayleigh scattering with the incident wavelength. As all the attenuation processes are dependent on wavelength, T_{L} is also dependent on air mass, although in a somewhat lesser manner than δ_{cda}. This variation of T_{L} for constant atmospheric clearness and the number of processes accounted for by T_{L} greatly hinders the practical utilization of Linke's formalism. This is illustrated in Fig. 1 for the 10th of August, 1997 in Geneva. We used broadband and spectral data acquired on a one-minute time step basis; the spectral measurements are taken with SolData sensors (Bason, 1997). On the left, the Kasten reviewed Linke turbidity T_{LK}:

$$T_{\mathrm{LK}} = \ln(I_o/B_n) \cdot (9.4 + 0.9 \cdot AM)/AM \tag{4}$$

is plotted versus time of day. The right graph is a Langley plot for the afternoon of that day and for $\lambda = 500$ nm. The linearity of the Langley plot indicates that the quantity of aerosols is relatively constant for the considered period in the afternoon. Ground measurements of the atmospheric humidity show also a good stability in the water vapor content of the atmosphere for the considered day. The vertical broken lines indicate an air mass equal to two.

In an attempt to improve the formulation, Louche *et al.* (1986) and Grenier *et al.* (1994, 1995) added absorption by the permanent gaseous constituents to the definition of δ_{cda} (these gases are considered uniformly mixed and invariable in both a clean dry atmosphere and a turbid atmosphere). Based on updated computed spectral data, Louche fitted a fourth order polynomial of the air mass to the optical thickness of a clean and dry atmosphere. Grenier, using a similar approach, added some minor changes to the spectral absorption and scattering equations yielding very similar values to Louche's relation:

$$\delta_{\mathrm{cda}} = (6.5567 + 1.7513 \cdot AM - 0.1202 \cdot AM^2 + 0.0065 \cdot AM^3 - 0.00013 \cdot AM^4)^{-1} \tag{5}$$

The resulting values of δ_{cda} are higher (and hence the resulting values of T_{L}, will be smaller) than those obtained with Kasten's pyrheliometric formula by as much as 25% for low values of air mass. Molineaux *et al.* (1995) noted that Louche and Grenier's expressions for δ_{cda} become divergent, respectively for air mass greater than 20 and 7. They adapted the coefficients of Linke's original expression to take into account the absorption by the permanent gases:

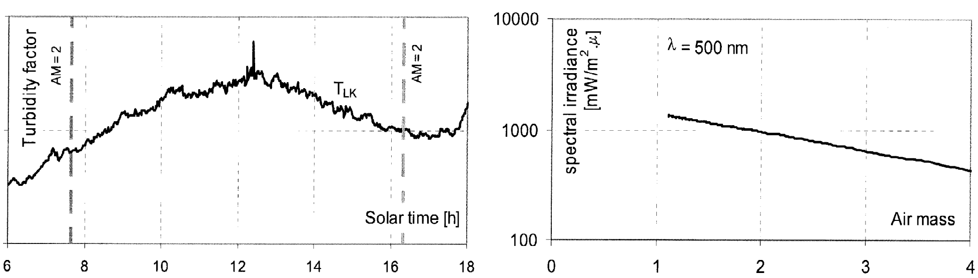

Fig. 1. One minute time step evolution of the Kasten modified Linke turbidity coefficient during the 10th of August 1997 in Geneva (left), and for the same day, the afternoon Langley plot for $\lambda = 500$ nm.

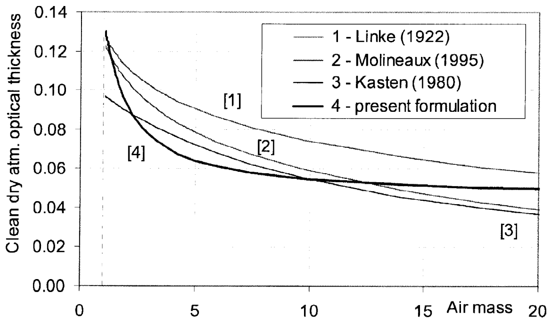

Fig. 2. Clean and dry optical depth based on theoretical considerations defined by Linke, Molineaux and Kasten, and our new formulation derived from measurements.

$$\delta_{\text{cda}} = 0.124 - 0.0656 \cdot \log(AM) \quad (6)$$

2.2. Present approach

Our approach in the determination of a new air mass-independent Linke turbidity coefficient is different. We did not attempt to produce a better formulation of δ_{cda}. Rather, we anchored our formulation on the Kasten-reviewed Linke turbidity T_{LK} at air mass 2 and considered it as the reference; therefore, at air mass 2, both Kasten's and our new expressions give the same turbidity value. For the beam and the global radiation components, we developed two empirical models that reproduce the observed daily variation of global and direct clear sky radiation measurements from seven environmentally distinct data banks. These models take into account the atmospheric turbidity and the altitude of the considered location. We then inverted the beam clear sky radiation model to extract the turbidity coefficient and the corresponding optical thickness δ_{cda}. Linke's, Kasten's and Molineaux optical thickness are based on theoretical considerations, whereas our new formulation is derived from a large set of measurements. The behavior of this clean and dry atmosphere optical thickness with the air mass is very different; it is empirical, based on experimental measurements and depending on physical parameters, it has no analytical correspondence to other definitions, but remains within the same limits as illustrated in Fig. 2.

3. EXPERIMENTAL DATA

We used data acquired at 7 stations with differing latitudes, altitudes and climates to develop our models; they are the following:

- Albany (NY), latitude 42.7°, longitude −73.9°, altitude 100 m, 1999
- Albuquerque (NM), latitude 35.1°, longitude −106.7°, altitude 1532 m, 1999
- Burns (OR), latitude 43.5°, longitude −119.0°, altitude 1265 m, 1999
- Burlington (KS), latitude 38.2°, longitude −95.6°, altitude 358 m, 1999
- Eugene (OR), latitude 44.1°, longitude −123.1°, altitude 150 m, 1999
- Geneva (Switzerland), latitude 46.2°, longitude 6.1°, altitude 420 m, mid 94–mid 95
- Hermiston (OR), latitude 45.8°, longitude −119.4°, altitude 180 m, 1999

Each of the data banks covers a full year of hourly data for the beam and the global radiation. The sites of Albuquerque, Albany and Burlington are either part of the ARM program (Stokes and Schwartz, 1994) or apply the stringent ARM calibration, characterization and quality check procedure. The quality control procedure developed in the frame of the international daylight measurements program within the International Commission on Illumination (CIE, 1994) was applied on the data acquired at the other stations.

4. TURBIDITY FACTOR AND NORMAL INCIDENCE BEAM CLEAR SKY MODEL

In 1983, Ineichen (1983) developed a clear sky radiation model based on 4 years of measurements taken in Geneva between 1978 and 1982. The initial model was fitted on 12 manually selected days and was of the following form:

$$B_{nc} = I_o \exp(-0.16 - 0.22 \cdot AM) \quad (7)$$

This early model assumed a constant Linke turbidity coefficient value of 3. When comparing this model against clear days manually selected from the 7 data banks, the diurnal shape was well respected, but, as would be expected, we observed seasonal/regional biases traceable to turbidity and altitude differences. We introduced the Linke turbidity coefficient T_{LK} at air mass 2, and a multiplicative coefficient b, depending on the altitude of the station. We obtained the following empirical expression for the normal beam clear sky radiation:[1]

$$B_{ncI} = b \cdot I_o \cdot \exp(-0.09 \cdot AM \cdot (T_{\text{LK}} - 1)) \quad (8)$$

[1]Further on in this paper, we describe a global radiation clear sky model. In order to remain coherent between the 3 radiation components, we apply a slight correction on the above model for very low turbidity conditions. The correction is given in Appendix A.

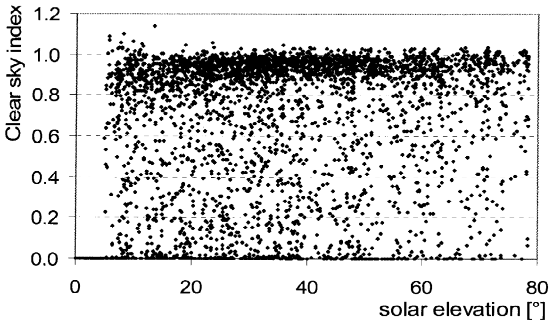

Fig. 3. Beam clear sky index $K_b = B_n/B_{ncI}$ versus solar elevation for the station of Albuquerque. The upper boundary is representative of clear conditions.

where $b = 0.664 + 0.163/f_{h1}$ ($f_{h1} = \exp(-altitude/8000)$ is taken from Kasten (1984), the altitude is expressed in meters). The seasonal trend of T_{LK} was evaluated with expression (4) at AM = 2 for clear sky conditions.

To illustrate the ability of the model to account for observed beam irradiance profiles, we represented in Fig. 3, the beam clear sky index $K_b = B_n/B_{ncI}$ versus solar elevation for the station of Albuquerque. The points near the upper boundary represent clear sky conditions; the figure shows a relative stability of the upper boundary with solar elevation, hence showing that the model adequately reproduces the clear sky air mass dependence of direct irradiance.

We then inverted the expression and extracted the turbidity factor:[2]

$$T_{LI} = [11.1 \cdot \ln(b \cdot I_o/B_{ncI})]/AM] + 1 \tag{9}$$

This new turbidity factor is represented in Fig. 4 (left) for measurements taken the 10th of August 1997 in Geneva in comparison with the Kasten modified Linke turbidity factor T_{LK}. The stability of the turbidity factor during the day is much better for the new coefficient. To further illustrate this point, we used SMARTS2 (Gueymard, 2001), a simple radiative transfer simulation model to evaluate the diurnal evolution of the two turbidity coefficients. The output of a radiative transfer model is not directly the turbidity coefficient. For a given data set of inputs, the model calculates the spectral radiation reaching the ground. By integration over the whole solar spectrum, it is then possible to evaluate the Linke turbidity factor by the use of Eq. (4), and the new turbidity factor with Eq. (9). To run the transfer model, we used a 5.7° circumsolar angle to replicate the aperture of standard direct irradiance measurements, we took the default values for O_2 and CO_2 absorption (taken from MODTRAN2 and SPECTRAL2 by Gueymard (2001)), 0.34 [atm-cm] for the ozone content, 0.0017 [atm-cm] for the NO_2, and 1.3 for the Angström α coefficient. For the water atmospheric content, we used 2.37 [pw-cm] derived from ground measurements of the temperature and relative humidity for that day. Considering Fig. 1, the turbidity is slightly different at air mass 2 in the morning and at same air mass in the evening. By iteration, using the transfer model, we extracted the Angström coefficient β for these 2 points and obtained 0.105 in the morning and 0.119 in the evening. We then evaluated the diurnal evolution of the two turbidity factors using the above method with a linear variation of the β coefficient during the day. The corresponding broken lines are plotted on the right graph; they are in good agreement with the measurements. A further illustration is given in Fig. 5 for clear day measurements at Albany, Burns, Eugene and Gladstone.

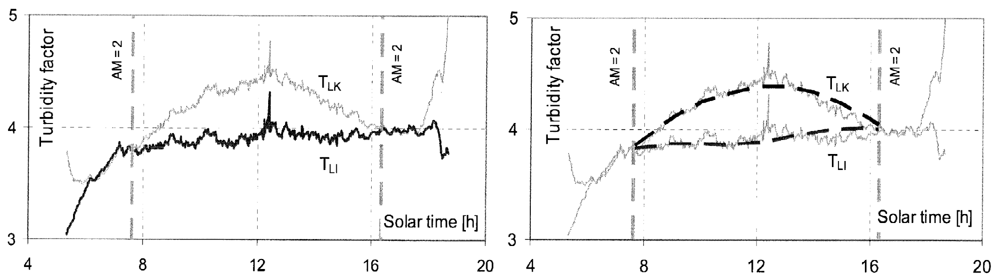

Fig. 4. Evolution of the two turbidity coefficients during the 10th of August 1997 in Geneva (T_{LK} = original Linke coefficient, T_{LI} = new Linke coefficient). On the right graph, we superimposed the corresponding values from the SMARTS2 simulations. The time step of the data is 1 min.

[2] In order to maintain coherence between the three global, direct and diffuse components for very low turbidity conditions, a small correction is added to this formulation as shown in Appendix A.

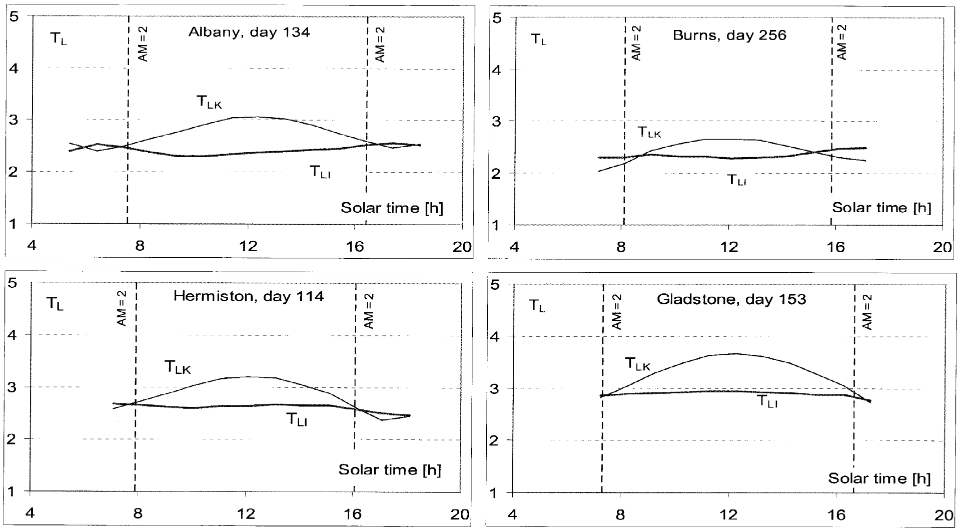

Fig. 5. Comparison of the new turbidity coefficient with T_{LK} for clear days at Albany, Burns, Hermiston and Gladstone.

Another possibility to illustrate the solar geometry stability of the new turbidity factor, is to represent hourly values versus solar altitude for all sky conditions. The lower boundary in the graphs represents clear sky conditions. In Fig. 6a, the original Kasten T_{LK} is represented for data acquired at the station of Burns. Values obtained with the corresponding new formulation T_{LI} are shown in Fig. 6b, T_{LI} is also given in Fig. 6c and 6d for the stations of Albany and Eugene.

5. GLOBAL AND DIFFUSE CLEAR SKY MODELS

We introduced these new models mainly for the readers' practical information, as a complement to the clear sky direct irradiance model.

Kasten (1984) proposed the following equation for the clear sky global radiation:

$$G_{hcK} = 0.84 \cdot I_o \cdot \sin(h) \cdot \exp(-0.027 \cdot AM \cdot (f_{h1} + f_{h2}(T_L - 1))) \quad (10)$$

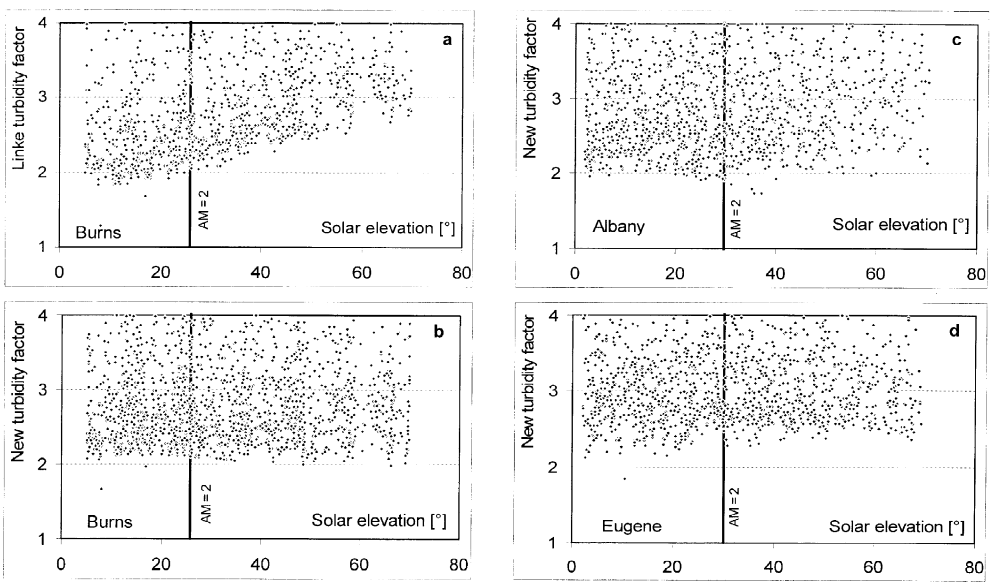

Fig. 6. Hourly values of the two turbidity factors versus solar elevation for the station of Burns, Oregon (a and b) and T_{LI} for Albany (c) and Eugene (d). The lower boundary is representative of winter clear sky conditions.

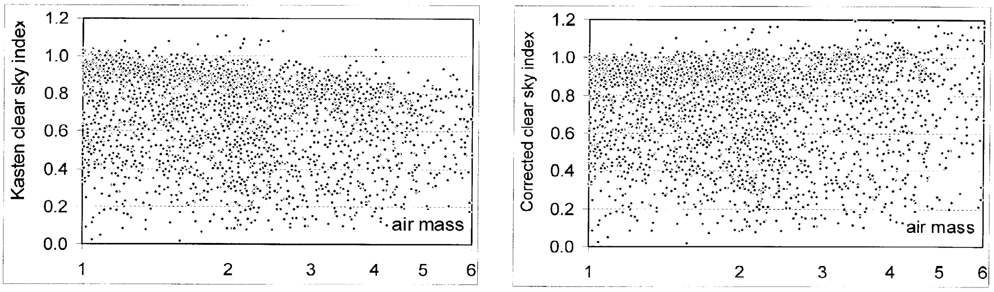

Fig. 7. Global clear sky index $K_c = G_h/G_{hc}$ versus air mass for the Linke turbidity coefficient T_{LK} (left) and the new coefficient (right). The decrease of the upper boundary on the left graph illustrates the model dependence with the air mass.

where G_{hcK} represents the global clear sky radiation reaching the ground on a horizontal surface, and f_{h1} and f_{h2} are coefficients (given in the nomenclature) that relate the altitude of the station with the altitude of the atmospheric interactions (Rayleigh and aerosols).

This formulation has the advantage to be adjustable for local/seasonal prevailing turbidity and site elevation; it was developed on data from Hamburg. When applying it to our set of data, we observed a solar elevation- (or air mass-) and a slight altitude-dependence of the model. We plotted in Fig. 7 the clear sky index $K_c = G_h/G_{hcK}$ versus the air mass on a logarithmic scale. The decrease with air mass of the upper boundary (clear conditions) on the left illustrates the air mass effect for data acquired at the station of Burns (altitude 1265 m).

With the help of specific days under particular clear sky conditions manually extracted from the different data sets, we included in the Kasten model two altitude dependent coefficients:

$$G_{hcI} = a_1 \cdot I_o \cdot \sin(h) \cdot \exp(-a_2 \cdot AM \cdot (f_{h1} + f_{h2} - (T_L - 1))) \quad (11)$$

where:

$$a_1 = 5.09.10^{-5} \cdot \text{altitude} + 0.868$$

$$a_2 = 3.92.10^{-5} \cdot \text{altitude} + 0.0387$$

with the altitude expressed in meters. The corrected model is shown on the right in Fig. 7. The diffuse clear sky radiation model is obtained by difference between the global and direct components, except under extreme conditions as noted in Appendix A.

6. CONCLUSION

The Linke turbidity coefficient T_L has been widely used since 1922 to characterize the degree of transparency of the atmosphere. This coefficient includes the effects of both aerosols and water vapor. T_L can be easily derived from broadband beam radiation measurements and therefore spatial and temporal maps can be derived from meteorological networks. The knowledge of the turbidity allows a good increase in the precision of solar radiation or daylight components evaluation in experimental applications.

The new formulation we propose has the advantages (1) to be solar altitude independent, and (2) to match to the original Linke turbidity factor at air mass 2—and therefore to remain coherent with the previous studies. It is based on high quality data representative of widely differing geographic locations, altitudes and climates.

In addition, we developed new formulations for clear sky global and direct irradiances taking into account the location's new Linke turbidity and altitude.

NOMENCLATURE

G_h	global horizontal radiation
G_{hc}	clear sky global horizontal radiation
G_{hcK}	Kasten clear sky global radiation model
G_{hcI}	new clear sky global radiation model
B_n	normal beam radiation
B_{nc}	clear sky normal beam radiation
B_{ncI}	new clear sky normal beam radiation model
D_h	diffuse horizontal radiation
δ_{cda}	clear and dry atmosphere optical thickness
T_L	Linke turbidity coefficient
T_{LK}	Linke turbidity coefficient corrected by Kasten
T_{LI}	new Linke turbidity coefficient
I_o	solar constant (Sun–Earth distance corrected)
AM	optical air mass
h	solar elevation angle
f_{h1}	exp(−altitude/8000)
f_{h2}	exp(−altitude/1250)

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APPENDIX A

When applying the global and beam clear sky models for very low turbidity conditions ($T_{LI} <$ 2), and to respect the coherence between the 3 radiation components, we apply the following empirical correction:

$$B_{ncI\,cor} = \min[B_{ncI}; G_{hc} \cdot \{1 - (0.1 - 0.2 \cdot \exp(-T_L))/(0.1 + 0.88/f_{h1})\}/\sin h]$$

For the same reasons, we slightly correct Linke turbidity factor for very low T_L:

$$T_{LI\,cor} = T_{LI} - 0.25 \cdot (2 - T_{LI})^{0.5}$$

The correction applies for $T_{LI} < 2$.

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